CREDIT PORTFOLIO OPTIMIZATION MODELS. ALGORITHMS FOR OPTIMAL CREDIT PORTFOLIO MANAGEMENT.

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INTRODUCTION

- In the last decades credit portfolio optimization has attracted a lot attention from both researchers and practitioners.

ROBUST MULTI-PERIOD PORTFOLIO MODEL

- The studies of behavioral portfolio theory show that investors could have numerous cognitive biases (e.g. mental accounting, loss aversion, etc.), which play important roles in decision-making process.

Problem Description

- Let there are one riskless asset a_0 and n risky assets $\{a_1, \ldots, a_n\}$ in security market for trading. An investor wants to make a multi-period investment strategy, where the investment duration is divided into T periods. Suppose that the investor holds a portfolio

$$X(t) = [x_{0,t}, x_{1,t}, \ldots, x_{n,t}]^T$$

- at time t, where $x_{0,t}$ denotes the wealth of riskless asset a_0 at time t, and $x_{i,t}$ denotes the wealth of risky asset a_i at time t, $i = 1, \ldots, n$, $t = 0, \ldots, T$.

the wealth of asset a_i at time *t* is

- Let $r_{0,t}$ and $r_{i,t}$ be the return of riskless asset a_0 and risky asset a_i at period *t* respectively,
  $i = 1, \ldots, n$, $t = 1, \ldots, T$.

- Then the wealth of asset a_i at time *t* is:

$$x_{i,t} = (r_{i,t} + 1) x^{+}_{i,t-1},\ i = 0, 1, \ldots, n, \quad t = 1, \ldots, T \qquad (1)$$

multi-period investment

- Using the recursive relationship in the multi-period investment, Eq. (1) can be rewritten as:

(2)

$$x_{i,t} = g_i(1,t)x_{i,0} + \sum_{j=1}^{t} g_i(j,t)\Delta x_{i,j-1},\ i = 0,1,\ldots,n,\ \ t = 1,\ldots,T;$$

- where $g_i(j,t)$ denotes the cumulative return of asset a_i from period j to period t,

$$g_i(j,t) = (r_{i,t} + 1)(r_{i,t-1} + 1)\ldots(r_{i,j} + 1),$$

$$g_i(t,t) = r_{i,t} + 1.$$

multi-period portfolio wealth

- From Eq. (2), the multi-period portfolio wealth at time *t* is given by

$$W_t = \sum_{i=0}^{n} x_{i,t} = \sum_{i=0}^{n} \sum_{j=1}^{t} g_i(j,t)\xi_{i,j-1}, \qquad t = 1,\ldots, T$$

(3)

$$\xi_{i,0} = x_{i,0} + \Delta x_{i,0},$$

$$\xi_{i,j} = \Delta x_{i,j}, \qquad i = 0,1,\ldots,n, \qquad j = 1,\ldots,T-1.$$

where

The robust optimization framework

- The robust optimization framework stipulates that only a subset of the uncertain coefficients will change and provides flexibility by adjusting the level of conservatism of the robust solution through the parameter Γ_t

the robust counterpart

- Following Eq. 4, the robust counterpart of is given as

$$(5)\ \max_{\{S_t \cup \{(v,d)\} | S_t \subseteq J_t, |S_t| = \lfloor \Gamma_t \rfloor, (v,d) \in J_t \setminus S_t\}} \left\{ \sum_{(i,j) \in S_t} \hat{g}_t(j,t) \left| \xi_{i,j-1} \right| + (\Gamma_t - \lfloor \Gamma_t \rfloor) \hat{g}_v(d,t) \left| \xi_{v,d-1} \right| \right\}$$

Eq. 5 shows that the conservatism of the solution is controlled by the parameter Γ_t

Conservatism of the solution

When $\Gamma_t = 0$ all uncertain returns are equal to $\bar{g}_i(j,t),\ i=1,\dots,n;\ j=1,\dots,t;$, then Eq. 5 is equivalent to the nominal problem.

When $\Gamma_t = n.t$, all uncertain returns realize the highest deviations, then Eq. 5 is equivalent to the worst-case problem.

We assume that the cumulative return takes values in the interval $[\bar{g}_i(j,t)-\hat{g}_i(j,t), \bar{g}_i(j,t)+\hat{g}_i(j,t)]$ where $\bar{g}_i(j,t)$ denotes the nominal value, and $\hat{g}_i(j,t)$ denotes the half-interval width of $g_i(j,t)$.

Formulation of the model

- Generally speaking, an investor's investment goal is to manage a portfolio in the manner that maximizes the PT value of the portfolio. More specifically, we quantify the total PT value as a weighted sum of PT value in each period. The objective function is expressed by

$$\max \sum_{t=1}^{T} \omega_t \times PV(W_t^R)$$

- where ω_t is the target weight at period t, $\omega_t \geq 0$.
- $t = 1,\ldots,T$.

FIREFLY – PATTERN SEARCH ALGORITHM

- Notice that model is a complex nonlinear programming problem. As a result, the traditional robust optimization techniques may fail to obtain the optimal solution.

- In order to solve the portfolio model effectively, we develop a hybrid Firefly-Pattern search algorithm

The Firefly algorithm

The firefly optimization (FFO) is a population-based stochastic optimization method first proposed by Yang (2008) to solve combinatorial / nonlinear optimiza-tion problems. FFO is inspired by the flight of fireflies and applies the following rules: a) each firefly is attracted only to the fireflies that are brighter than itself; Strength of the attractiveness is proportional to the firefly's brightness, which attenuates over the distance; the brightest firefly moves randomly, and b) brightness of every firefly is determined by its solution quality.

In the FFO, there are two important issues: the variation of light intensity and formulation of the attractiveness. Here the attractiveness of a firefly is determined by its brightness which in turn is associated with the objective function value. The light intensity varies according to the inverse square law.

Pattern search

Pattern search (PS) is a family of numerical optimization methods that do not require the gradient of the problem to be optimized. Hence PS can be used on functions that are not continuous or differentiable. Such optimization methods are also known as direct-search, derivative-free, or black-box methods.

Pattern search

The name "pattern search", was coined by Hooke and Jeeves (1961)

One theoretical parameter (variable) is varied at a time by steps of the same magnitude, and when no such increase or decrease in any one parameter further improved the fit to the experimental data, the step size is halved and the process is repeated until the steps were deemed sufficiently small. On this base is developed the generalized pattern search (GPS) algorithm.

CONCLUSION

- Firstly, considering the uncertainty of security returns, we employ the robust optimization approach of Bertsimas et al. to formulate a robust multi-period portfolio, which introduces a parameter to indirectly adjust the degree of conservatism of the robust solutions.
- Second, a dynamic prospect theory value function is used, where we modify the loss aversion parameters, which are updated dynamically.

THANK YOU FOR
THE KIND ATTENTION!