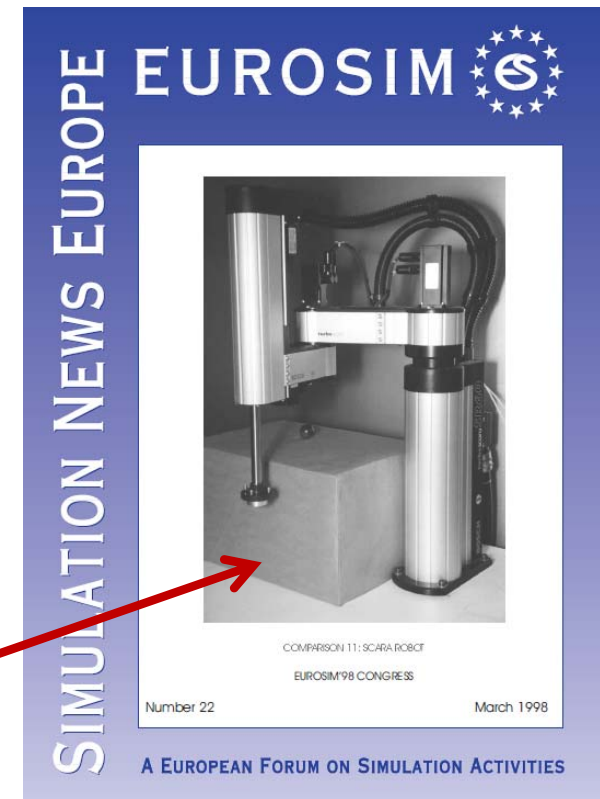


# **Multi-method Modelling and Simulation and Co-Simulation - Development, Trends and Education**

Felix Breitenecker, Andeas Körner, Nikolas Popper  
Mathematical Modelling and Simulation Group  
Institute for Analysis and Scientific Computing,  
Vienna Univ. of Technology, Austria

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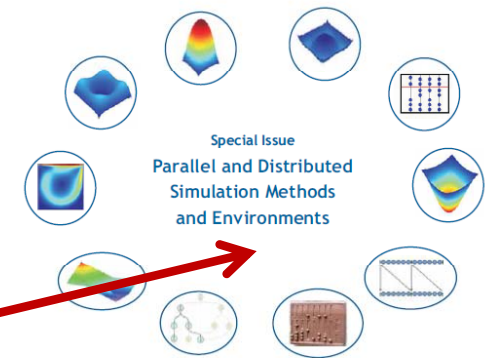
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- C16 Restaurant Business Dynamics, SNE 14(1), 2004
- C17 Spatial Dynamics of SIR Epidemics, SNE 14(2-3), 2004;
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- C19 Pollution in Groundwater Flow, SNE 15(2-3), 2005
- CP2 Parallel & Distributed Simulation, SNE 16(2), 2006



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C18 Neural Networks vs. Transfer Functions, SNE 15(1), 2005

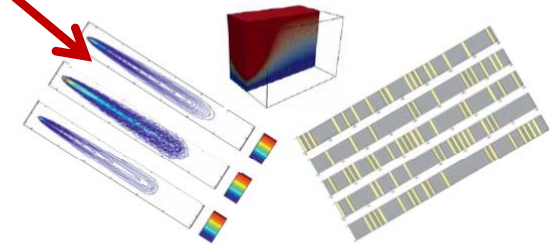
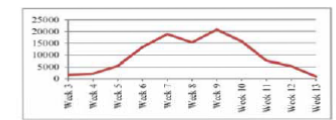
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- Extended solution documentation (> 1 page)
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- Documentation and publication in SNE of 'solutions' may take more pages – up to 10 pages SNE.

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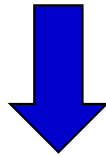
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## DEVELOPMENT OF SYSTEM SIMULATION

$$\dot{\vec{x}}(t) = \vec{f}(t, \vec{x}(t), \vec{u}(t)), \quad \vec{x}(t_0) = \vec{x}_0, \quad \text{ODE}$$



$$\begin{aligned} \dot{\vec{x}}(t) &= \vec{f}(t, \vec{x}(t), \vec{z}(t), \vec{u}(t), \vec{p}), & \vec{x}(t_0) &= \vec{x}_0 \\ \vec{g}(\vec{x}(t), \vec{z}(t), \vec{u}(t), \vec{p}) &= \vec{0} \end{aligned} \quad \text{DAE}$$

$$h^B(\vec{x}(t), \vec{u}(t), \vec{p}) \stackrel{! \pm}{=} \vec{0} \Rightarrow E^B(\vec{x}(\hat{t}^B), \vec{u}(\hat{t}^B), \vec{p})$$

...

## State Events

## DEVELOPMENT OF SYSTEM SIMULATION

$$\dot{\vec{x}}(t) = \vec{f}(t, \vec{x}(t), \vec{z}(t), \vec{u}(t), \vec{p}), \quad \vec{x}(t_0) = x_0$$

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...

- Parameter change event – SE-P
- Input change event – SE-I
- State change event – SE-X
- Function change event – SE-F
- Structure change event – SE-S
- Output trace event – SE-O
- Algorithm event – SE-A

**DAE**

**State Events**

**Event function**

**Event action**

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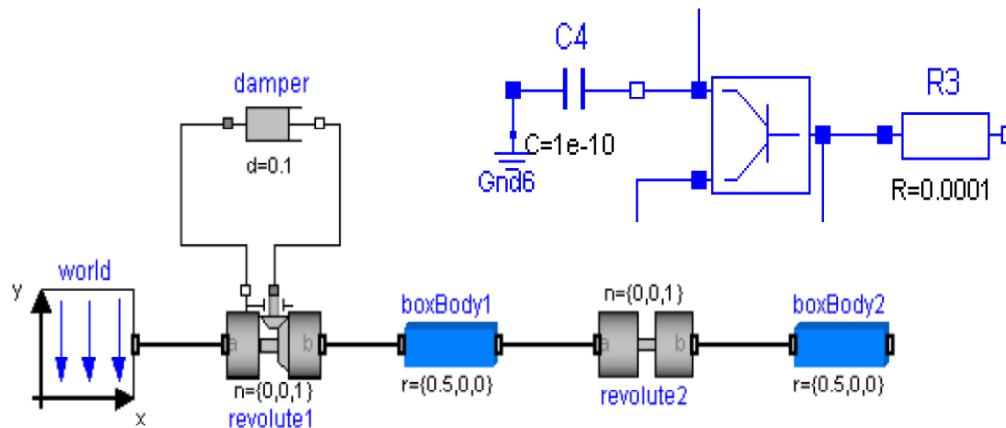
...

**DAE**

**State Events**

**Event function**

**Event action**



**Physical Modelling – State Space ,unknown‘**



## STATE EVENT HANDLING

$$\dot{\vec{x}}(t) = \vec{f}(t, \vec{x}(t), \vec{z}(t), \vec{u}(t), \vec{p}), \quad \vec{x}(t_0) = x_0$$

$$\vec{g}(\vec{x}(t), \vec{z}(t), \vec{u}(t), \vec{p}) = \vec{0}$$

**DAE**

$$h^B(\vec{x}(t), \vec{u}(t), \vec{p}) \stackrel{! \pm}{=} \vec{0} \Rightarrow E^B(\vec{x}(\hat{t}^B), \vec{u}(\hat{t}^B), \vec{p})$$

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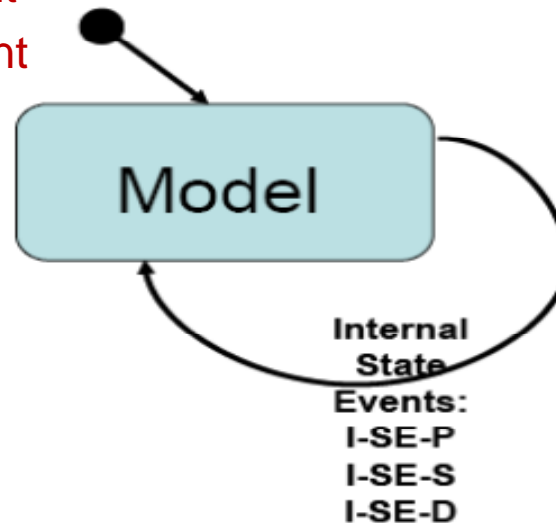
**Structural-dynamic  
Systems**

# Structural-dynamic Systems

$$h^B(\vec{x}(t), \vec{u}(t), \vec{p}) \stackrel{! \pm}{=} \vec{0} \Rightarrow E^B(\vec{x}(\hat{t}^B), \vec{u}(\hat{t}^B), \vec{p})$$

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- Parameter change event –
- Input change event
- State change event
- **Function change event**
- **Structure change event**
- Output trace event
- **Algorithm event**



1. *Detection* of the event
2. *Localisation* of event
3. **Event Action**
4. *Restart* of solver

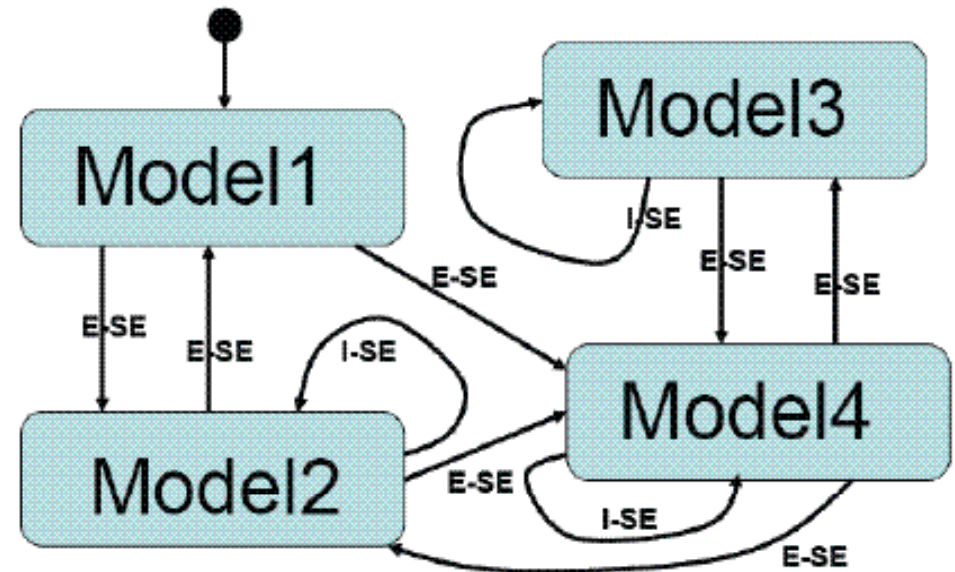
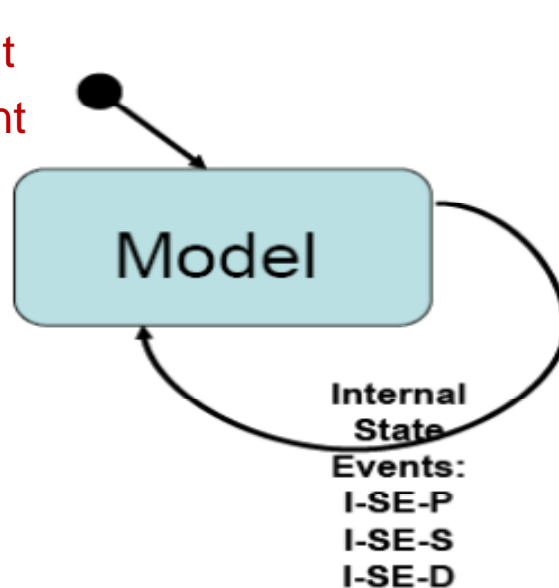
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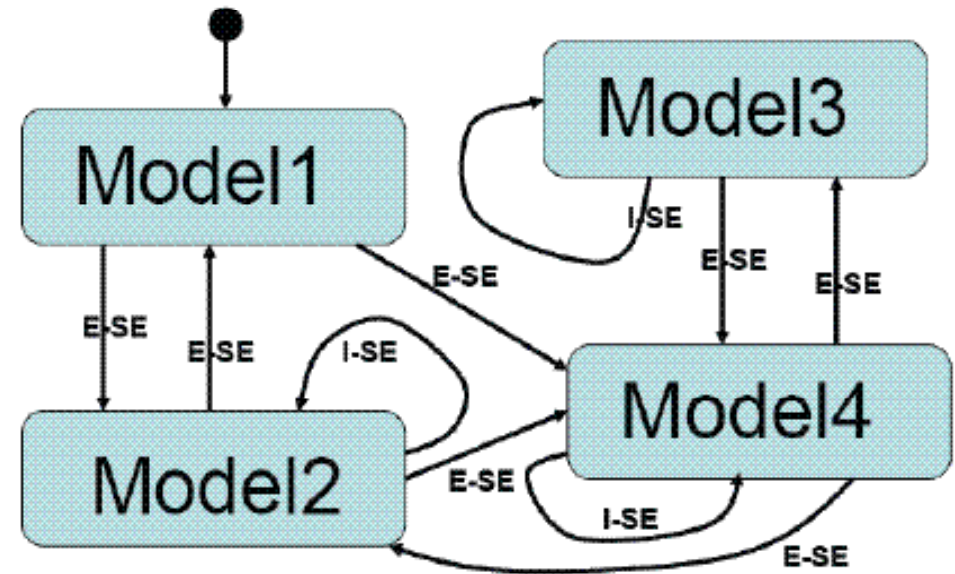
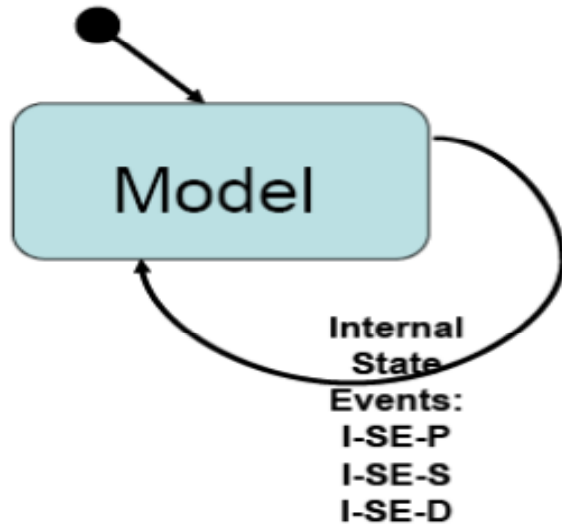
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# Multimethod Modelling



# Multimethod Modelling

DAEs

Diff Eq.

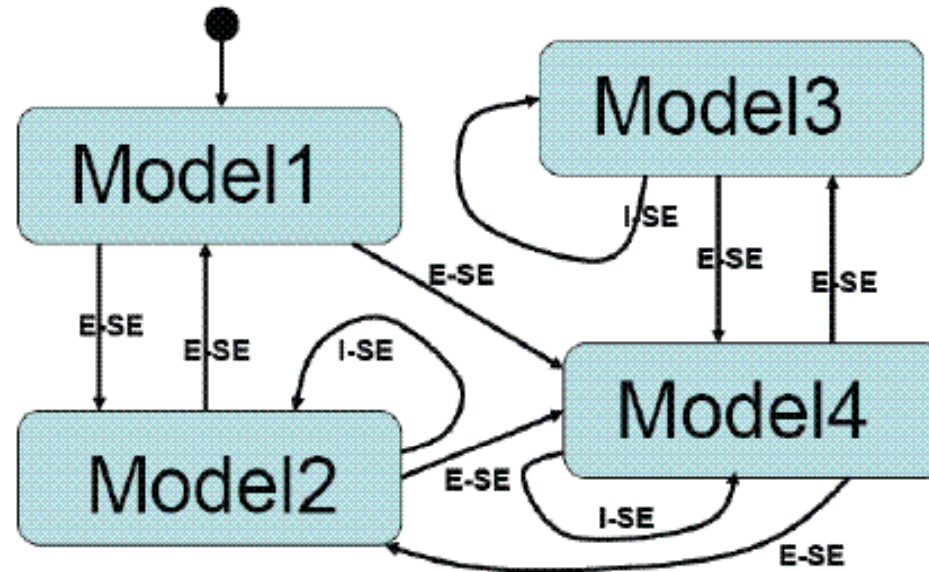
PDEs

Event Mod

DEVS

Process Mod

Agent-based



Charact. Functions

Static Mod

# Multimethod Modelling

**DAEs**

Diff Eq.

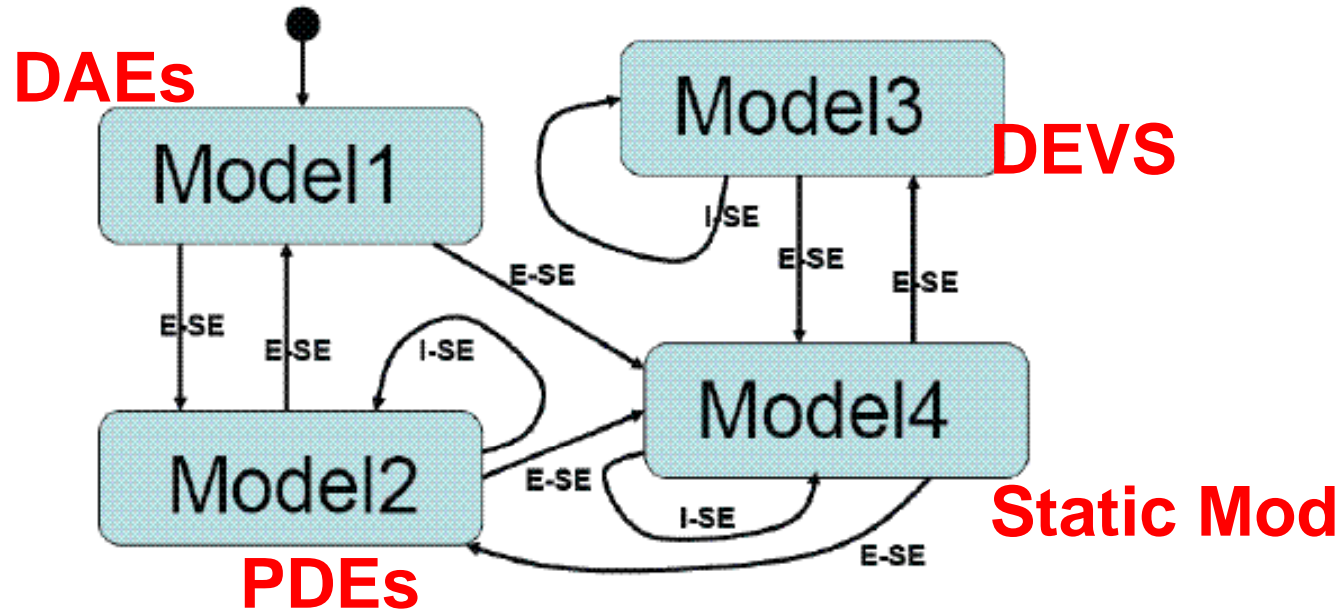
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**Static Mod**

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**Diff Eq.**

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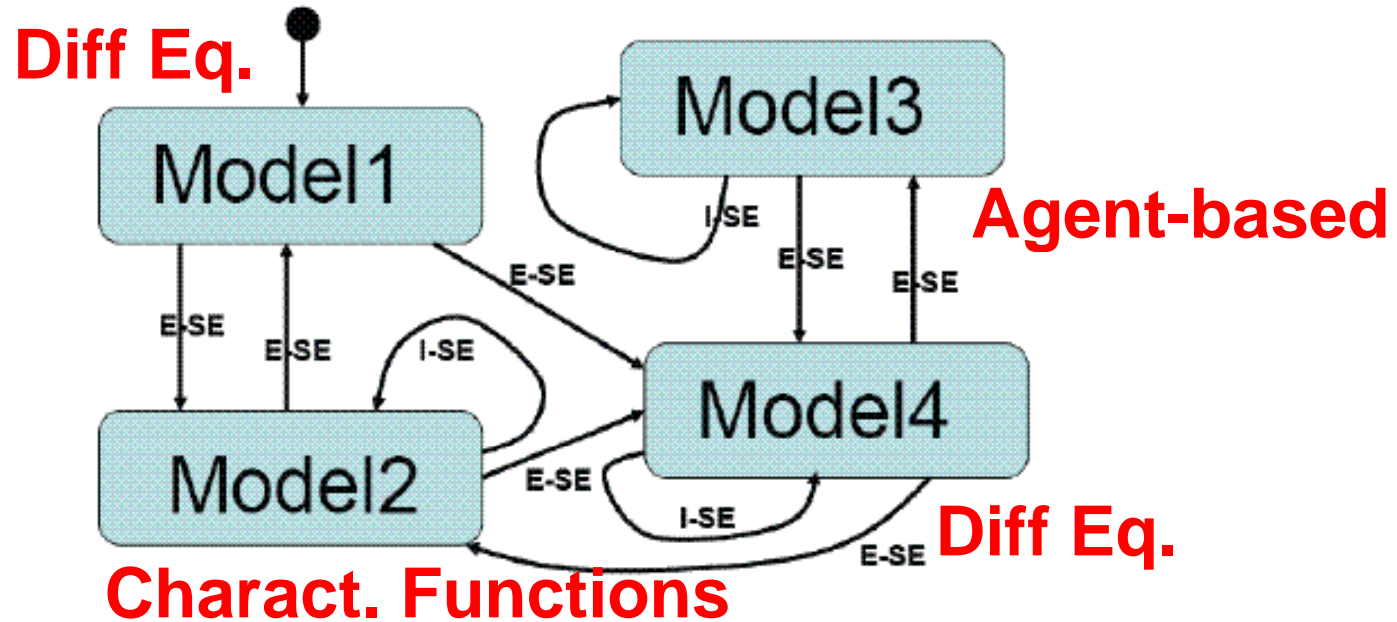
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Static Mod



# Multimethod Modelling

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Diff Eq.

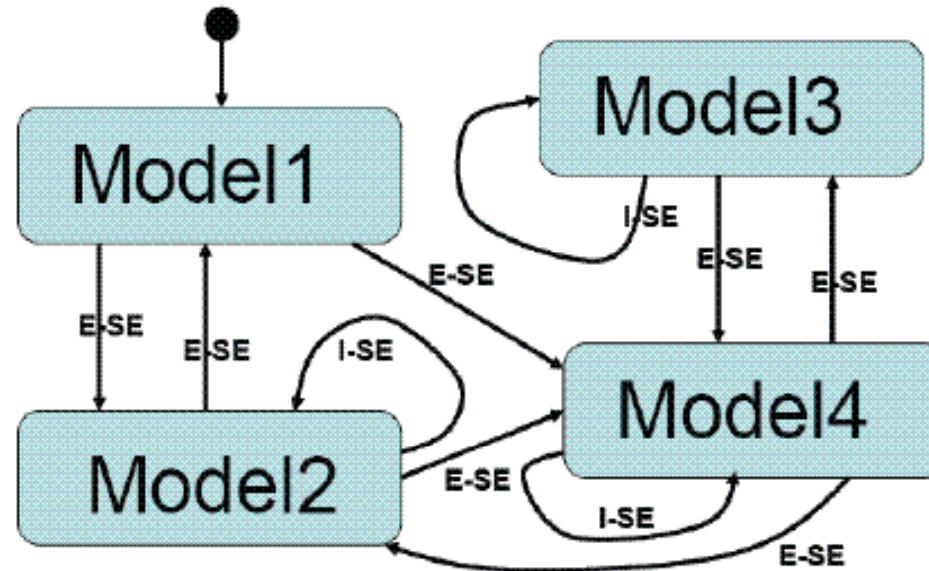
PDEs

Event Mod

DEVS

Process Mod

Agent-based



Transfer Functions

System Dynamics

Modelica

Bond Graphs

Agent-based

Petri Nets

CA Mod

State Automata

Charact. Functions

Static Mod



# Multimethod Modelling

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Diff Eq.

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Event Mod

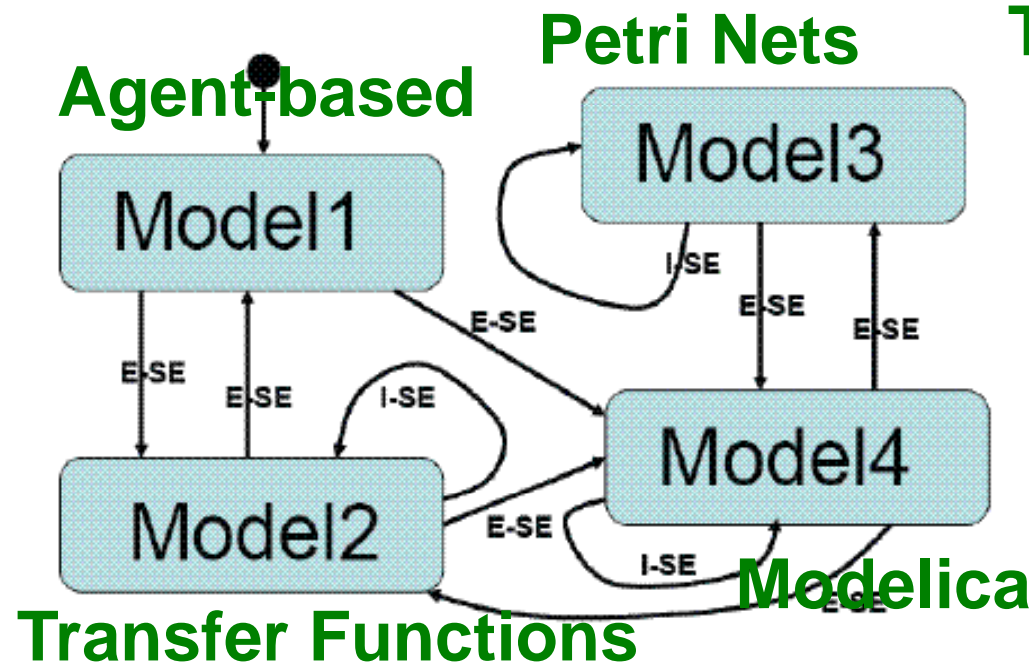
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State Automata

# Multimethod Modelling

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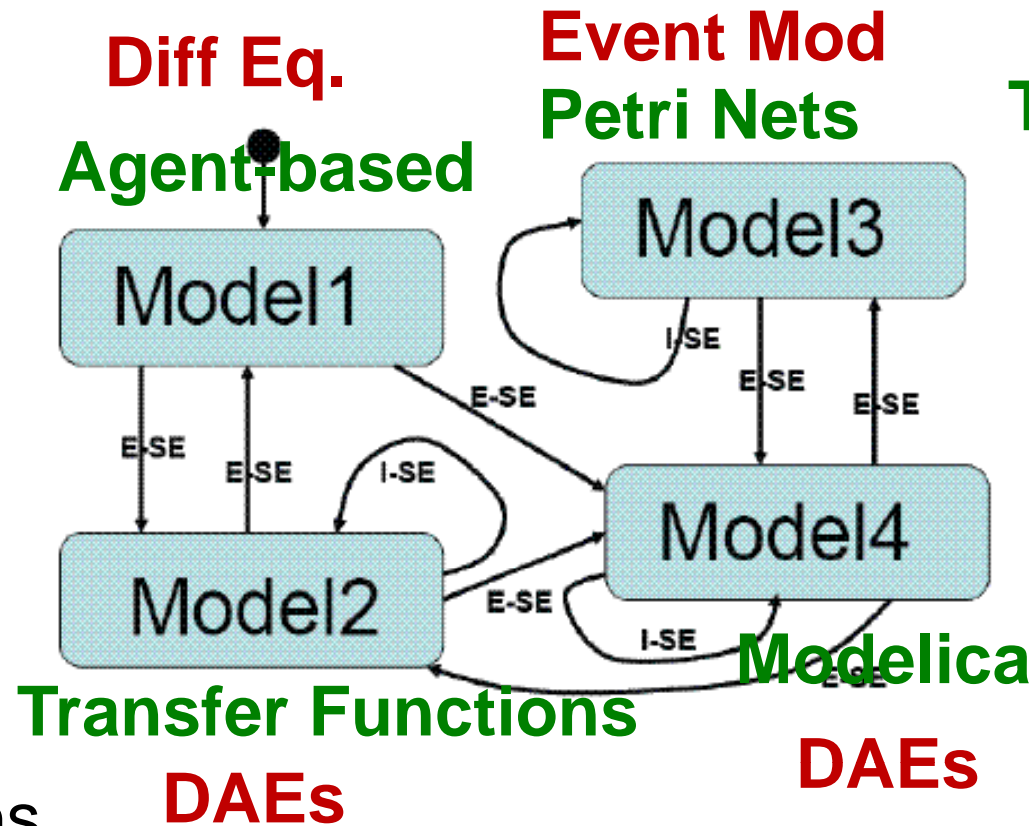
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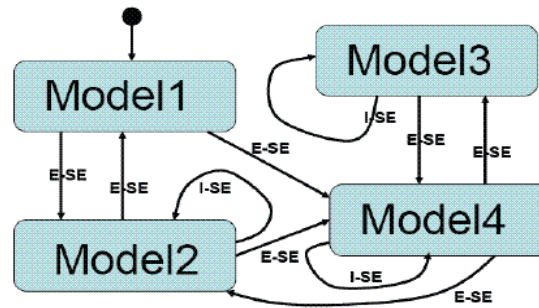
CA Mod

State Automata

# Multimethod Modelling

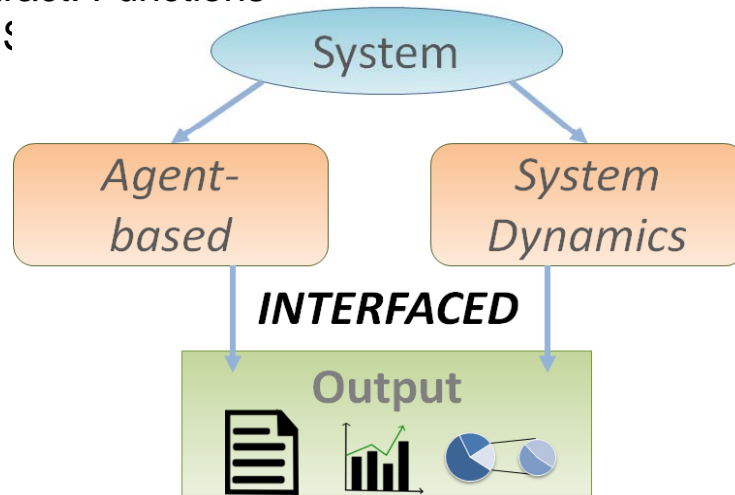
DAEs  
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PDEs

Event Mod  
DEVS  
Process Mod  
Agent-based



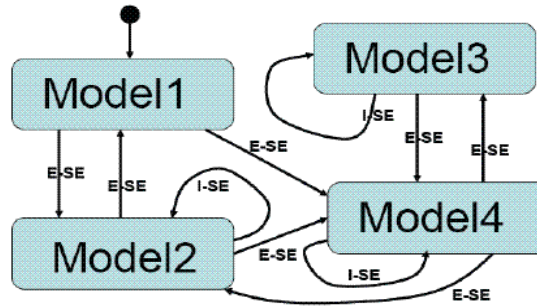
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{



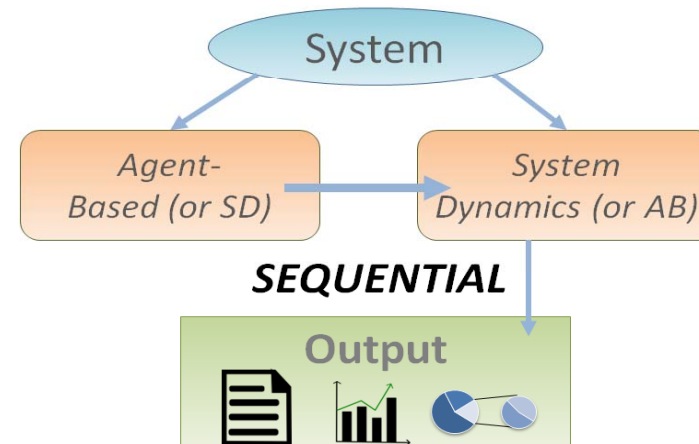
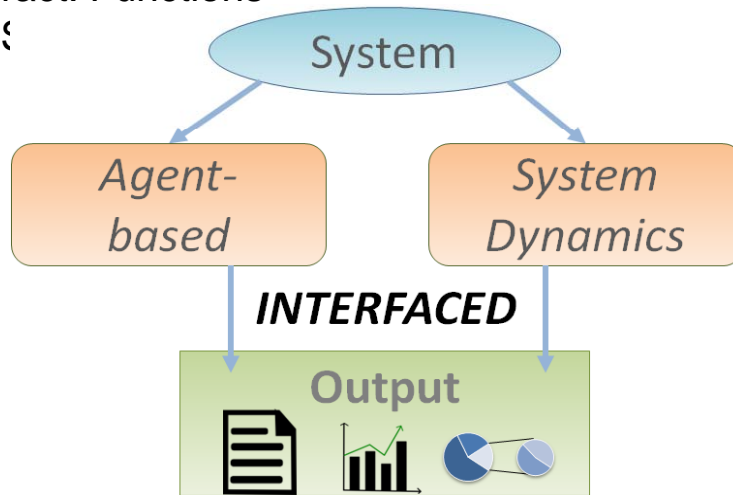
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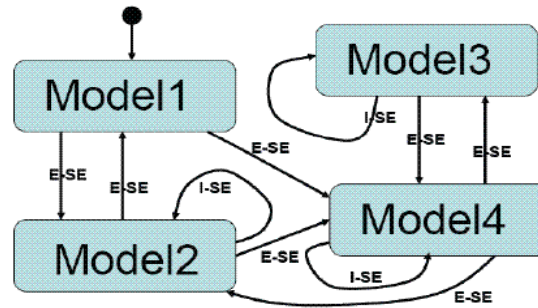
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# Multimethod Modelling

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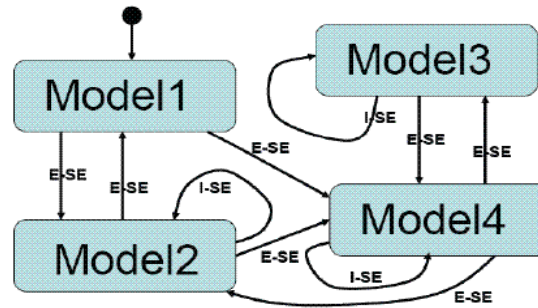


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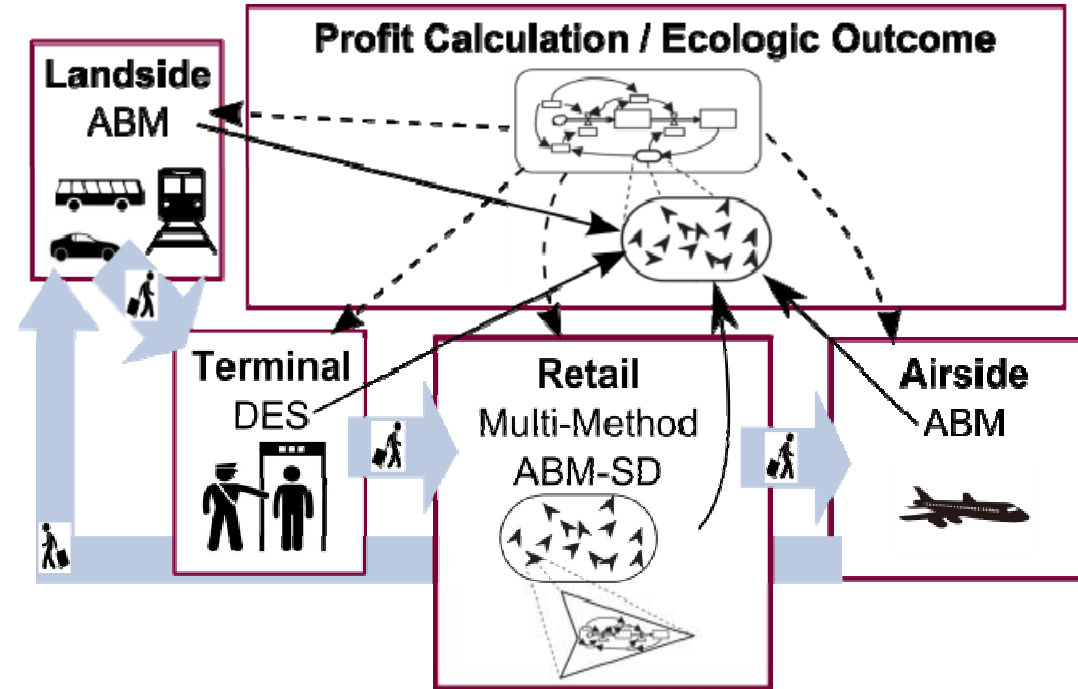


# Multimethod Modelling

- DAEs
- Diff Eq.
- PDEs
- Event Mod
- DEVS
- Process Mod
- Agent-based
- Charact. Functions
- Static Mod



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# Multimethod Modelling and CoSim

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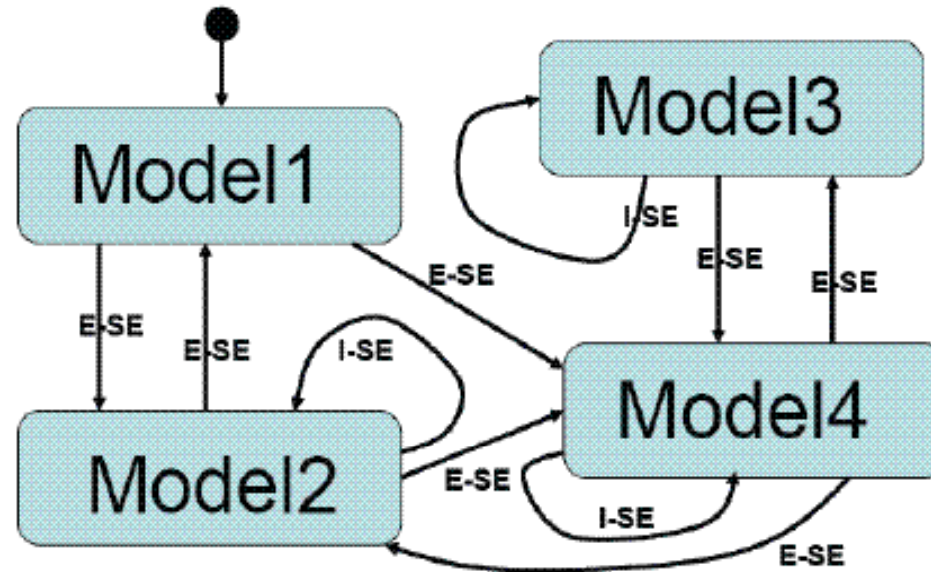
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## Consequences for Edu in ModSim ?

# Multimethod Modelling and CoSim

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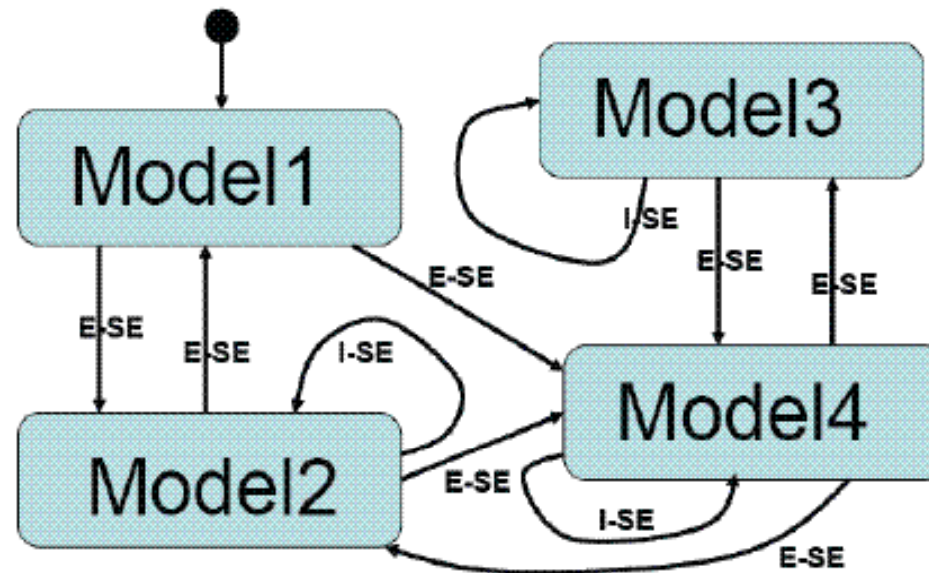
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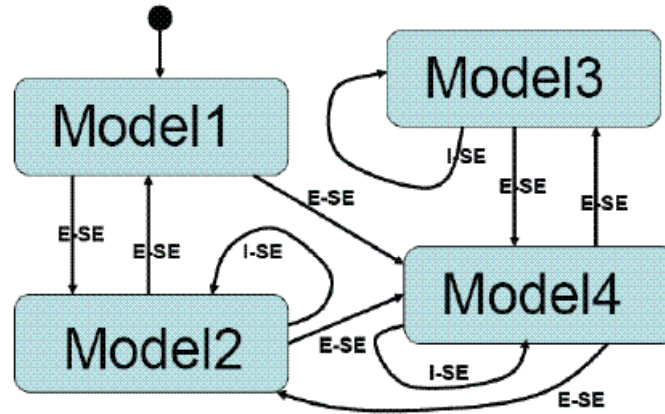
## Consequences for Edu in ModSim ?

### Challenge



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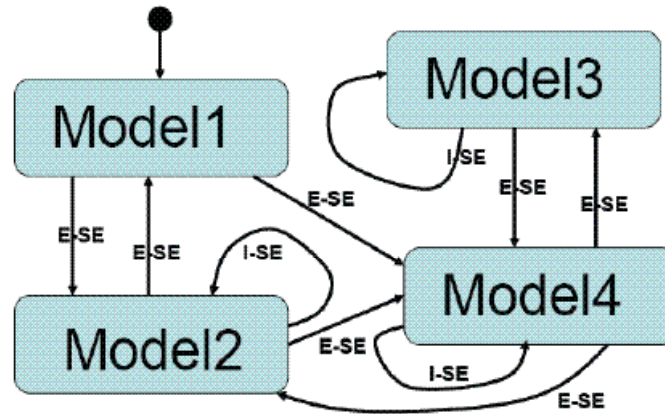
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## -Interdisciplinary Edu in ModSim

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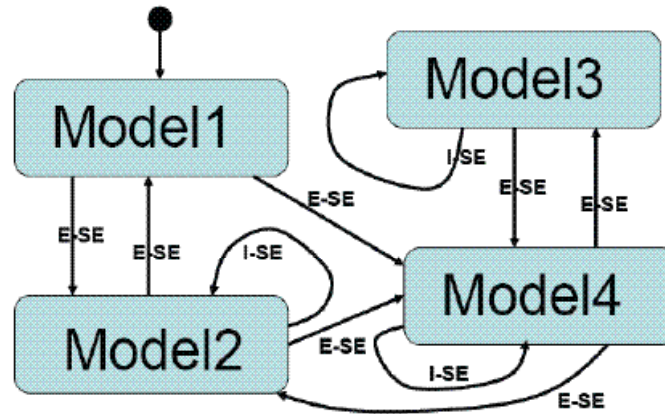
### Challenge

- Interdisciplinary Edu in ModSim
- Basic Method Edu in ModSim
- Basic Tool Edu in ModSim

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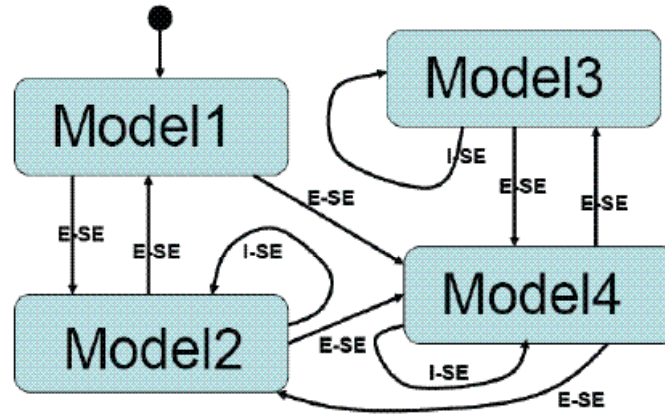
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### Challenge

- Interdisciplinary Edu in ModSim
- Basic Method Edu in ModSim
- Basic Tool Edu in ModSim
- Specialisation Edu in ModSim

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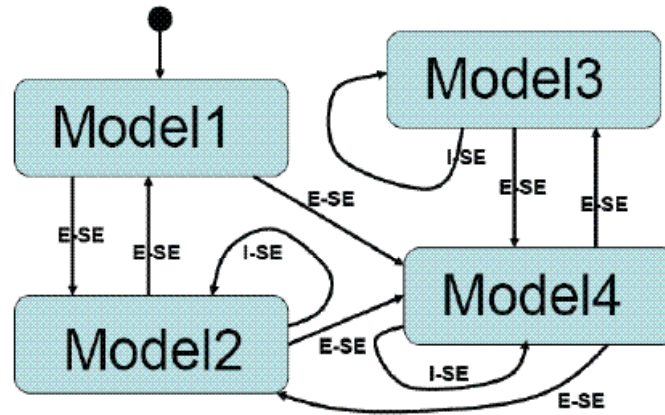
**Challenge**

**Possible Solutions**

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- Specialisation Edu in ModSim

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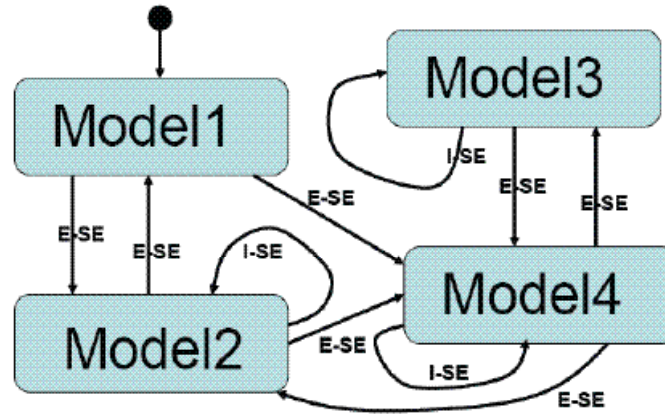
**Possible Solutions**

- Interdisciplinary Edu in ModSim In existing or new curricula
- Basic Method Edu in ModSim
- Basic Tool Edu in ModSim
- Specialisation Edu in ModSim

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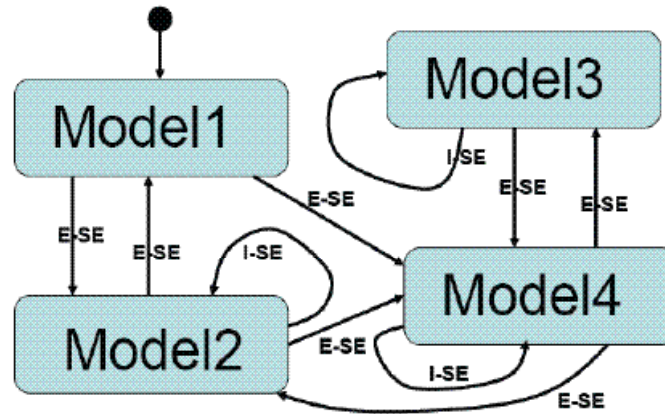
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- Interdisciplinary Edu in ModSim
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In existing or new curricula  
 In pref. ,more' Univ. Sites

# Multimethod Modelling and CoSim

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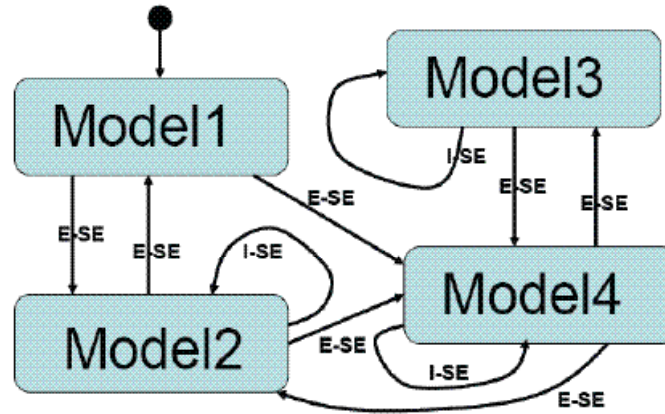
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- In existing or new curricula
- In pref. ,more' Univ. Sites
- Student Exchange

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## Consequences for Edu in ModSim ?

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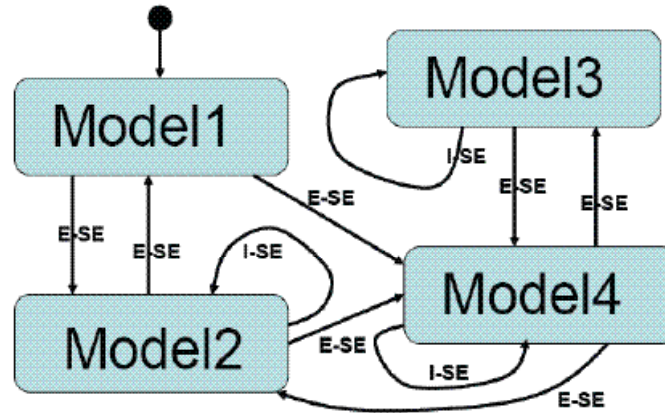
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- Lecturer Exchange



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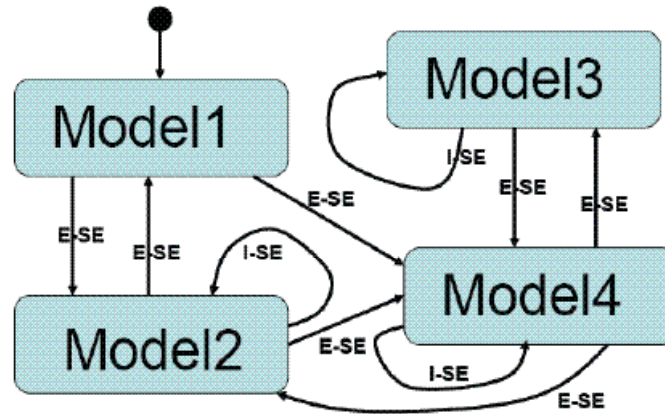
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- Basic Method Edu in ModSim
- Basic Tool Edu in ModSim
- Specialisation Edu in ModSim

- In existing or new curricula
- In pref. ,more' Univ. Sites
- Student Exchange
- Lecture(r) Exchange

# Multimethod Modelling and CoSim

DAEs  
Diff Eq.  
PDEs

Event Mod  
DEVS  
Process Mod  
Agent-based



Transfer Functions  
System Dynamics  
Modelica  
Bond Graphs  
Agent-based  
Petri Nets  
CA Mod  
State Automata

Charact. Functions  
Static Mod

## Consequences for Edu in ModSim ?

### Challenge

### Possible Solutions

- |   |  |
|---|--|
| <ul style="list-style-type: none"> <li>-Interdisciplinary Edu in ModSim</li> <li>-Basic Method Edu in ModSim</li> <li>-Basic Tool Edu in ModSim</li> <li>-Specialisation Edu in ModSim</li> </ul> | <ul style="list-style-type: none"> <li>In existing or new curricula</li> <li>In pref. ,more' Univ. Sites</li> <li>Student Exchange</li> <li>Lecturer Exchange</li> <li>Lecture Exchange - EL,DL</li> </ul> |
|---|--|

# Multimethod Modelling and CoSim

*Thank you for Attention  
On good Cooperation!*

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### Possible Solutions

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