



PÉCSI TUDOMÁNYEGYETEM  
♦ JUBILEUM 650 ♦  
UNIVERSITY OF PECS JUBILEE



Computer-aided Courses of Mathematics  
at University of Pécs,  
Faculty of Engineering and Information Technology

Ildikó Perjési-Hámori  
University of Pécs, Hungary



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Hungarian King Louis the Great initiated establishment of a university in the episcopal city of Pécs in 1367 Fresco of Andor Dudás in the Hall of University of Pécs (1923)



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**Faculty of Engineering and  
Information Technology**  
Architecture, civil- , environmental-  
electrical engineering, information  
technology and architectural design  
BSc, MSc, DLA, PhD,  
English, Hungarian





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## Research groups

- Energy Design Research Team
- Efficiency of Resources Research Team
- Building Energetics and Building Ecology Research Team
- Sustainable Cities Research Team
- Heritage Protection Research Team
- ***Computer Algebraic and Dynamic Geometrical Systems in Higher Education***
- Structural Diagnostics and Analysis Research Team
- Solidarity Architecture Research Team
- Virtual Measuring Systems and Machine Perception

PhD – DLA Symposium <https://phdsymp.mik.pte.hu/>

Doctoral students present their research topics and results at an international conference. The International PhD Symposium has been held annually for 11 years and over the two days of the symposium an estimated 160 presentations are delivered in different sections.



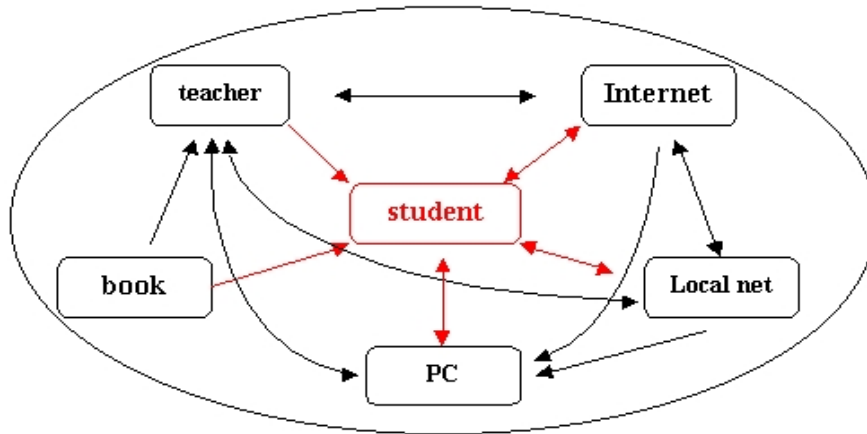
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## History of our research team

- **Experimental period** – first steps – new tool – revolution – new didactical access (computer algebra system - CAS - was new for teachers and students)
- **Discovering period** – pre-designed worksheets – usage as many times as possible (new for students)
- **Period of expanded use** – new didactical tasks – limits of utility – development of hardware and software – test and assessment systems – many CAS applications for mobile phone – integrating programming, engineering and Math courses (it is the part of every day life)
- **Future ?**



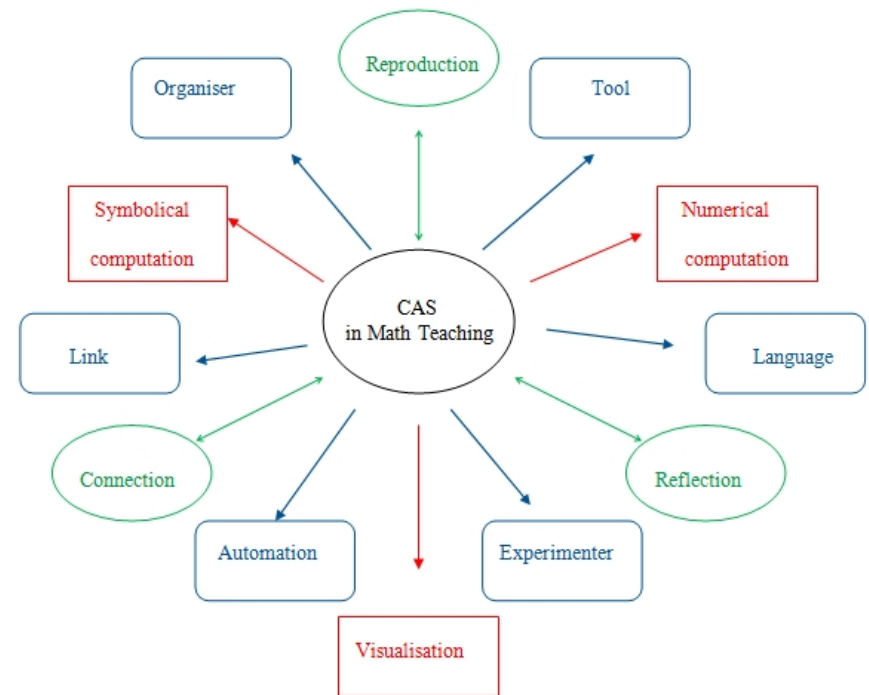
## Experimental period



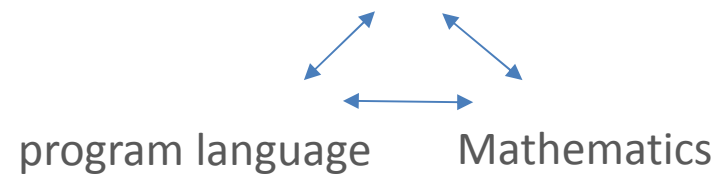
Interactive learning environment

### Difficulties:

- Inexperience
- Technical hadness 1D input
- Lack of time
- Didactical problems



common language





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## Experimental period

Expectations and observations:

- Students can become active participants in the learning-teaching process
- Using the tools makes it possible to teach concepts which are often used in engineering
- Extend creative learning
- Structured knowledge-building – modularization
- Multiple representation
- Changing learning style of students passive  $\Rightarrow$  active, concrete  $\Rightarrow$  abstract
- Development of conjecture
- Easy visualization



From the Newton-law:

> `eq:=diff(x(t),t$2)+2*beta*diff(x(t),t)+omega^2*x(t)=0;`

$$eq := \left( \frac{d^2}{dt^2} x(t) \right) + 2 \beta \left( \frac{d}{dt} x(t) \right) + \omega^2 x(t) = 0$$

It's seen that this equation for  $x(t)$  is a second-order, constant-coefficient, linear, homogenous differential equation system.

> `gen_sol:=dsolve(eq,x(t));`

$$gen\_sol := x(t) = \_C1 e^{((- \beta + \sqrt{\beta^2 - \omega^2}) t)} + \_C2 e^{((- \beta - \sqrt{\beta^2 - \omega^2}) t)}$$

Let's check the shape of the given path-time function! Find the particular solution when  $x(0)=0$ ,  $D(x)(0)=vmax$ .

> `part_sol:=dsolve({eq,x(0)=0,D(x)(0)=vmax},x(t));`

$$part\_sol := x(t) = \frac{1}{2} \frac{vmax e^{((- \beta + \sqrt{\beta^2 - \omega^2}) t)}}{\sqrt{\beta^2 - \omega^2}} - \frac{1}{2} \frac{vmax e^{((- \beta - \sqrt{\beta^2 - \omega^2}) t)}}{\sqrt{\beta^2 - \omega^2}}$$

### Instrumental orchestration

- Have short textual explanations
- Introduce first every new command - according to the principle of spirality - through a mathematical problem





```
> restart;with(VectorCalculus):  
> SetCoordinates('cartesian'[x,y,z]);
```

*cartesian*<sub>x,y,z</sub>

```
> field:=VectorField(<x,-z,2*z>);
```

*field* := (x)*e*<sub>x</sub> - z*e*<sub>y</sub> + 2z*e*<sub>z</sub>

```
> curve:=<r*cos(t),r*sin(t),t*h/(2*Pi)>;
```

*curve* := (r cos(t))*e*<sub>x</sub> + (r sin(t))*e*<sub>y</sub> +  $\frac{1}{2} \frac{t h}{\pi} e_z$

```
> LineInt(field,Path(curve,t=0..2*Pi),'inert')=LineInt(field,Path(curve,t=0..2*Pi));
```

$$\int_0^{2\pi} \left( -r^2 \cos(t) \sin(t) - \frac{1}{2} \frac{t h r \cos(t)}{\pi} + \frac{1}{2} \frac{t h^2}{\pi^2} \right) dt = h^2$$

```
> Lineintegral:=proc(mezo,gorbe,t1,t2)
```

```
  local erinto,skalarszorzat,integrandus,integral,lok;
```

```
  erinto:=diff(gorbe,t);
```

```
  lok:=subs({x=gorbe[1],y=gorbe[2],z=gorbe[3]},mezo);
```

```
  skalarszorzat:=DotProduct(erinto,lok);
```

```
  integrandus:=simplify(skalarszorzat);
```

```
  integral:=Int(integrandus,t=t1..t2)=int(integrandus,t=t1..t2);
```

```
end;
```

```
> Lineintegral(field,curve,0,2*Pi);
```

$$\int_0^{2\pi} \left( -\frac{1}{2} \frac{2r^2 \cos(t) \sin(t) \pi^2 + t h r \cos(t) \pi - t h^2}{\pi^2} \right) dt = h^2$$

### Instrumental orchestration

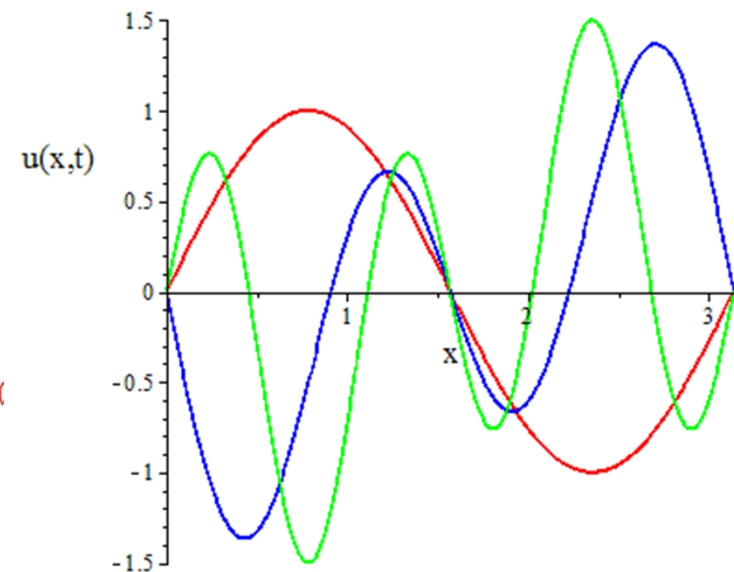
- Step by step  $\Rightarrow$  self made procedures  $\Rightarrow$  built in procedures



## Discovering period

*Black box: if we have no time, or students have only few knowledge of the solution of the differential equation, and Fourier series*  
*White box: after the detailed explanation, for experiments, or solving more complicated problem*

```
> restart;with(VectorCalculus):with(plots):  
> Cord_Eqn := { diff( u(x,t), t, t) = Laplacian( u(x,t), 'cartesian'[x] )};  
Cord_Eqn := {  $\frac{\partial^2}{\partial t^2} u(x,t) = \frac{\partial^2}{\partial x^2} u(x,t)$  }  
> sol:-animate( [[u(x,t) , color=red],  
                [-u(x,t)/2-sin(4*x) , color=blue],[-u(x,t)/2+sin(6*x) ,color=green]], t=(  
                labels=["x", "u(x,t)"], labelfont=[TIMES,ROMAN,14],  
                scaling=constrained);
```



Instrumental orchestration

Black box white box: visualization, engineering applications



### Expectations and observations

- Network based learning
- Cover the whole syllabus of the course
- Usage as many times as possible
- Instrumental orchestration
  - Have short textual explanations
  - Introduce first every new command - according to the principle of spirality - through a mathematical problem
  - Applications being written according with gradation (white box – black box)
  - Step by step  $\Rightarrow$  self made procedures  $\Rightarrow$  built in procedures
  - Black box white box: visualization, engineering applications

### Difficulties:

- Deep understanding only for the best students
- Everything is ready: no conceptual understanding
- Didactical problems:
  - Some exercises became routine ones with help of it
  - Not the technical details but the mathematical meaning is always the most important
  - Avoid using CAS only for the end in itself; it is only the inferior of the mathematical subject matter



$\int f dx$	$\int_a^b f dx$	$\sum_{i=k}^n f$
$\prod_{i=k}^n f$	$\frac{d}{dx} f$	$\frac{\partial}{\partial x} f$
$\lim_x f$	$a+b$	$a-b$
$a \cdot b$	$\frac{a}{b}$	$a^b$
$a_n$	$a_*$	$\sqrt{a}$
$\sqrt[n]{a}$	$a!$	$ a $
$e^a$	$\ln(a)$	
$\log_{10}(a)$	$\log_b(a)$	
$\sin(a)$	$\cos(a)$	$\tan(a)$
$\binom{a}{b}$	$f(a)$	$f(a,b)$
	$f:=a \rightarrow y$	
	$f:=(a,b) \rightarrow z$	
$f(x) _{x=a}$	$\begin{cases} -x & x < a \\ x & x \geq a \end{cases}$	

Printed:  $\int_2^3 \frac{\sin x}{\cos^2 x} + \sqrt[3]{x} dx$

>  $\int_2^3 \frac{\sin(x)}{\cos(x)^2} + \sqrt[3]{x} dx$

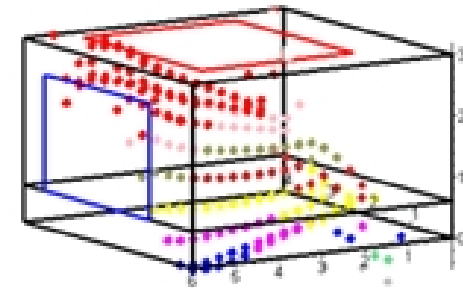
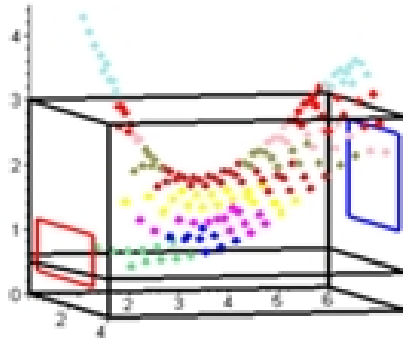
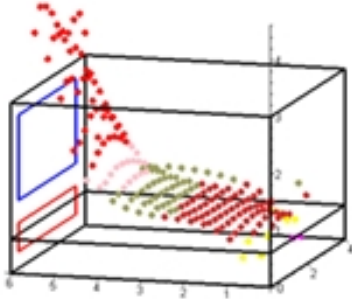
> `int(sin(x)/cos(x)^2+x^(1/3), x = 2 .. 3);`

>

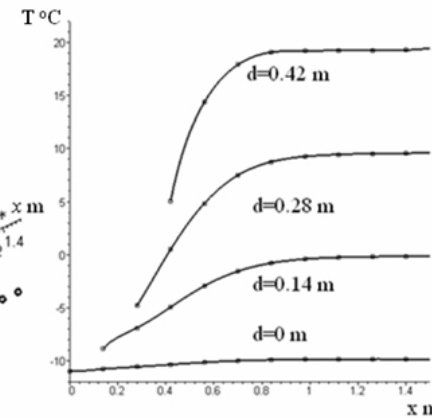
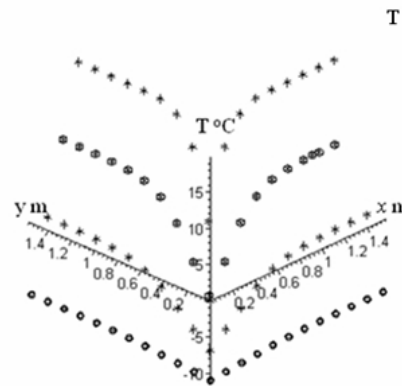
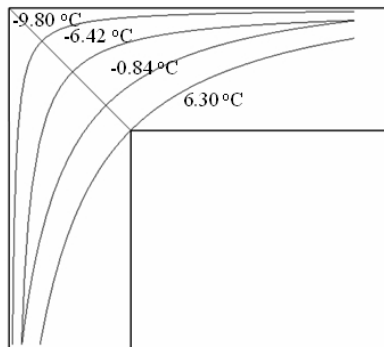
User friendly interface: it is not necessary to prepare everything

**Lecture:** presentation (ppt, Prezi, video...) definition, theorems, few example + oral explanation

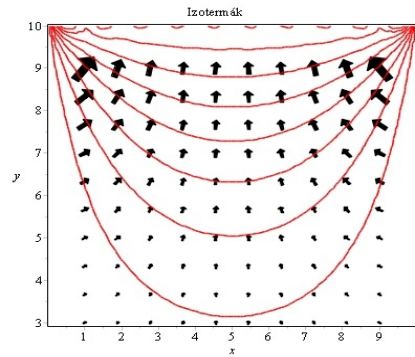
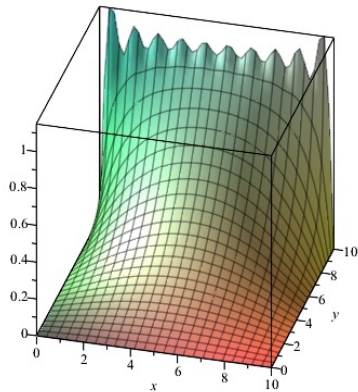
**Seminar:** paper work, simple examples , more complicated examples using CAS, independent student work



I. Perjési- Hámori: Simulation of Heat Radiation Asymmetry With Maple 7th Vienna Conference on Mathematics Modelling Febr. 15-17, 2012



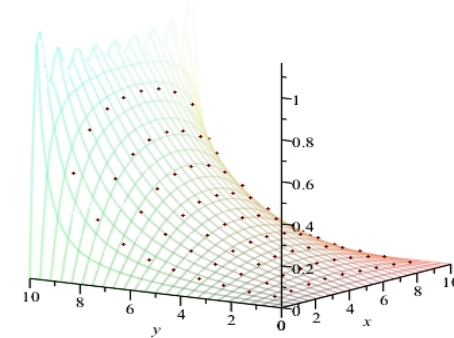
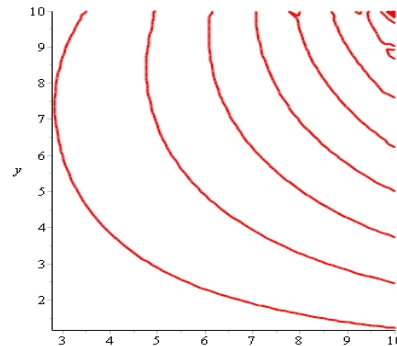
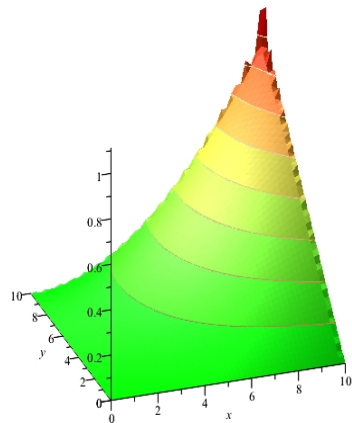
J. Vajda, I. Perjési-Hámori: *Two dimensional mathematical model of heat-transmission of one- and double-layer building* Pollack Periodica Vol. 2, No.3, pp.25-34, 2007.



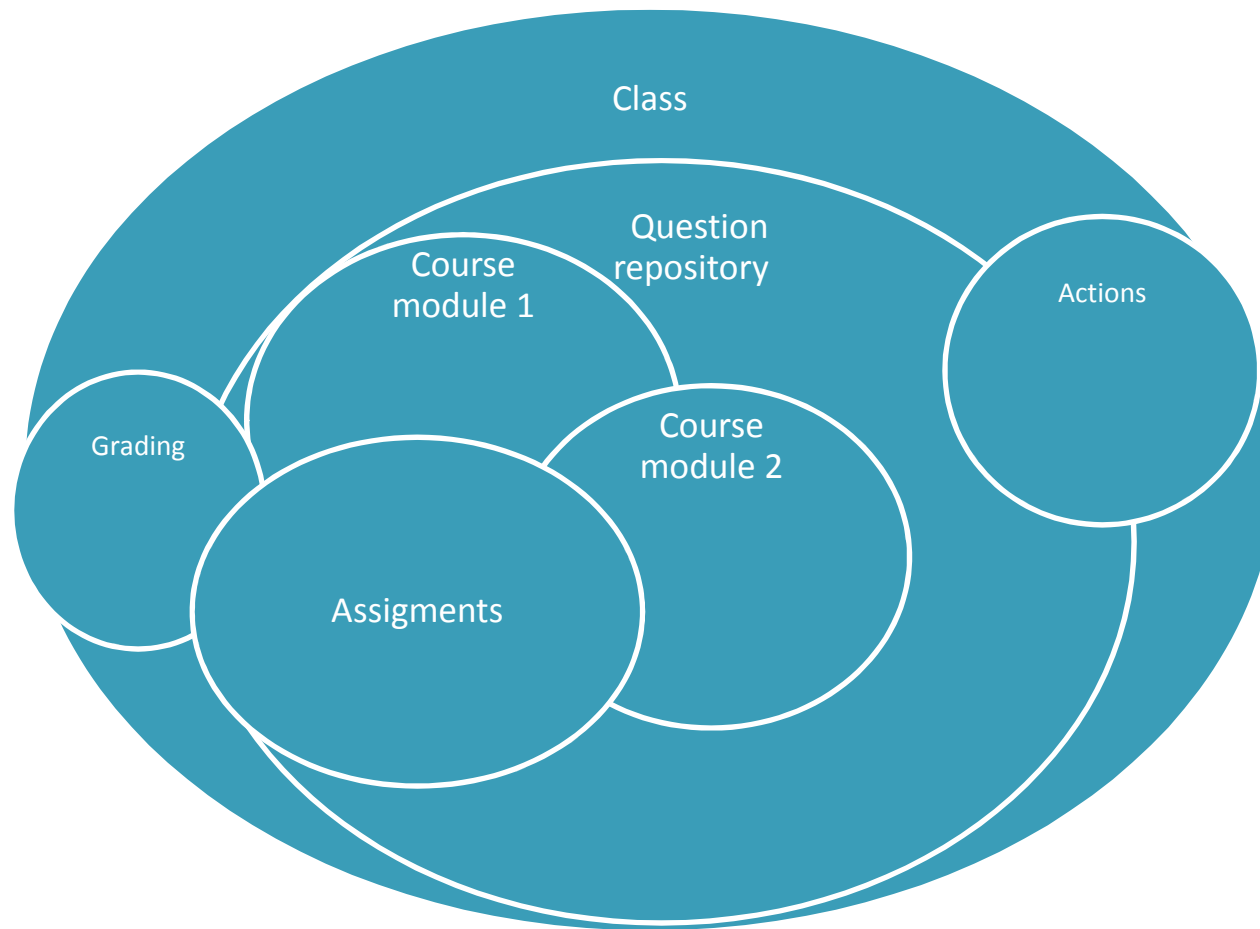
```

> elliptic := proc (n, m, a, b, c, d, f, g)
local i, j;
global l, e, u, h, k;
for j to m-1 do
for i to n-1 do
l := (j-1)*(m-1) + i;
h := (b-a)/n; k := (d-c)/m;
if l mod (n-1) = 1 then u[l-1] := g(a, c+j*k) fi;
if l mod (n-1) = 0 then u[l+1] := g(b, c+j*k) fi;
if l-n+1 <= 0 then u[l-n+1] := g(a+i*h, c) fi;
if l+n-1 > (n-1)*(m-1) then u[l+n-1] := g(a+i*h, d) fi;
e[l] := k^2*(u[l-1]-2*u[l]+u[l+1]) + h^2*(u[l-(n-1)]-2*u[l]+u[l
+ (n-1)]) = f(a+i*h, c+j*k);
unassign('u[l-1]','u[l]','u[l+1]','u[l-n+1]','u[l+n-1]');
od;
od;
end proc;

```



I. Perjési-Hámori: Two Dimensional Mathematical Model of Heat-transmission Using MAPLE poster 8th Vienna Conference on Mathematics Modelling Febr. 17-20, 2015 Mathematical Modelling , Volume # 8 | Part# 1 689-690





## Matematika3

PTE

Ildikó Perjesiné Hámori ([perjesi@pmmik.pte.hu](mailto:perjesi@pmmik.pte.hu))

Select the link for an assignment to begin:

Assignment Name	Points	Type	Availability
<a href="#">Kétváltozós integrál - feladat</a>	24.0	Homework/Quiz	Unlimited
<a href="#">Kétváltozós integrál elmélet</a>	8.0	Homework/Quiz	Unlimited
<a href="#">Kétváltozós függvény gradiens és szélsőérték- feladatok</a>	31.0	Homework/Quiz	Unlimited
<a href="#">Kétváltozós függvény parciális és iránymenti derivált-feladatok</a>	32.0	Homework/Quiz	Unlimited
<a href="#">Kétváltozós függvények deriválása-elmélet</a>	5.0	Homework/Quiz	Unlimited
<a href="#">Függvénysor elmélet</a>	10.0	Homework/Quiz	Unlimited
<a href="#">Függvénysorok gyakorló feladatok</a>	12.0	Homework/Quiz	Unlimited
<a href="#">Szamsorok elmelet gyakorlo</a>	10.0	Homework/Quiz	Unlimited
<a href="#">Szamsoros feladatok gyakorlo</a>	30.0	Homework/Quiz	After 4/26/13 9:42 AM

### Question 4: (1 points)

A  $T$  tartományon integrálható  $f(x, y)$  kétváltozós függvény  $t(T) \neq 0$  területű  $T$  tartományra vonatkozó integrálközepértékén az  $\frac{1}{t(T)} \cdot \iint_T f(x, y) dx dy$  kifejezéssel definiált számot értjük.

Határozza meg az  $f(x, y) = x + 2y + 2$  függvény  $T$  tartományra vonatkozó integrálközepértékét, ha a tartományt az  $x$ -tengely, az  $x = 4$  egyenes és a  $g(x) = 2\sqrt{x}$  függvény grafikonja határolja.

$t(T) =$

A belső integrál értéke:

A kettős integrál értéke:

Az integrálközepérték:





Question 2: (1 points)

Válassza ki az alábbi tartományok közül azokat, amelyek esetén az  $\iint_T f(x, y) dT$  integrál az  $f(x, y) = x^2 + 4y$  függvény grafikonja és a  $T$  tartomány által határolt hengeres térrész térfogatának számértékét adja.

- $-2 \leq x \leq 2$  és  $-1 \leq y \leq 1$
- $-2 \leq x \leq 2$  és  $0 \leq y \leq 1$
- $y = \sqrt{x}$  és  $y = 2x - 1$
- $y = 4x^2$  és  $y = x + 5$  görbék által határolt tartomány.

How did I do?

Comment:

Your response

Válassza ki az alábbi tartományok közül azokat,

amelyek esetén az  $\iint_T f(x, y) dT$  integrál az

$f(x, y) = x^2 + 4y$  függvény grafikonja és a  $T$  tartomány által határolt hengeres térrész térfogatának számértékét adja.

0% (0%)

Correct response

Válassza ki az alábbi tartományok közül azokat,

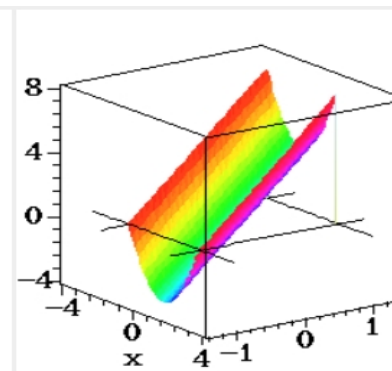
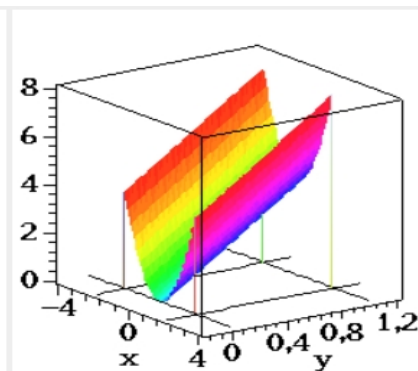
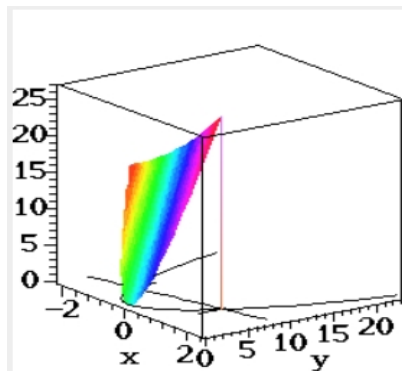
amelyek esetén az  $\iint_T f(x, y) dT$  integrál az

$f(x, y) = x^2 + 4y$  függvény grafikonja és a  $T$  tartomány által határolt hengeres térrész térfogatának számértékét adja.

$-2 \leq x \leq 2$  és  $0 \leq y \leq 1$ ,  $y = 4x^2$  és  $y = x + 5$  görbék által határolt tartomány.



Incorrect





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Period of expanded use

### Expectations and observations:

- Students are aboriginals, teachers are immigrants in IT
- Students are users but do not know about programming
- User friendly interface
- Test and assessment system based on Maple
- CAS applications for mobile phone, free software's (GeoGebra)
- Integrating programming, engineering and math courses (it is the part of every day life)

### Difficulties:

- Didactical problems:
- Role of teacher is not clear
- Why we have to understand math, why is not enough the applications?
- There are standards in the softwares, which one is the more useful? (Price, university licenses, comparison)



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## Conclusion

No way to ignore software or own-written computer programs in mathematics and engineering education and research.

In Hungary:

Secondary school: e.g. GeoGebra (Geomatech project), Euklides, Cindarella

University level: basic courses: e.g. MAPLE, Matematika

special courses: e.g. Matlab, Autocad, Archicad, ANSYS, COSMOS,



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**Thank you for your attention**

Colleagues and students are welcome in Pécs