MINIMIZATION OF VOTING RESULTS DISTORTION. OPTIMIZATION METHODS.

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INTRODUCTION

Elections are a natural part of all contemporary democratic societies. In this form we know them they represent an inseparable part of the society's governing since 12th century.

Throughout the centuries a lot of different methods for conduct of elections have been developed. Those include the method of voting, of calculation of elections' results, of elections' winner determination. Elections are widely carried out both for sole authority bodies' selection (mayor, president, prime minister) and for collective authority bodies' selection (committees, municipalities' councils and parliament).

The elections' three main functions are as follows:

- To ensure representation;
- To determine the policy carried out and to ensure investiture;
- To legitimate the elected one's power.

PROBLEM DEFINITION

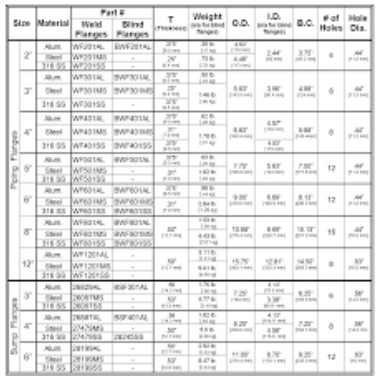
The implementation of the so defined three major tasks of the elections, especially the first one ensuring representation, is a direct function of the punctuality and accuracy of the procedures embedded in the basis of elections process.

Unfortunately there is no electoral system that ensures absolute representation of the elections' results. All electoral systems in existence are based on ensuring that the results shall approach to the actual electoral state.

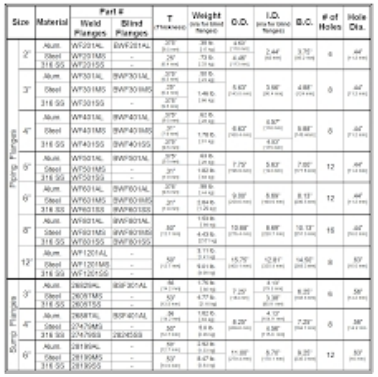
The distortions of electoral results may be defined as the variations between the actual state and the representation derived from the elections' results.

PROBLEM DEFINITION

Votes
integers



Distribution
floating point



Mandates
integers

FACTORS: ELECTORAL SYSTEMS

There are several various electoral systems – means of voting and results definition. The choosing the electoral system is among the major factors that influence the distortions.

- **Majoritarian electoral systems**
- **Semi-proportional Systems**
- **Proportional Representation Systems**
- **Mixed systems**

- **The causes of distortion**

FACTORS: CALCULATION PROCEDURES

Using the proportional electoral systems requires usage of calculation procedures – mathematical methods through which the final results are obtained. As it is not possible to get a part of a mandate, those calculation procedures represent problems for transformation of floating-point values (proportions from a mandate) into integer values (mandates).

- **Highest averages methods**
  - D'Hondt
  - Sainte-Laguë
- **Methods of the largest remainder**
  - Hare Niemeyer
  - Droop quota

The cause of distortion in the election results lies down in the various approaches in the rounding of the values obtained in the calculation procedures.

FACTORS: CALCULATION PROCEDURES

Highest Averages Methods, in which the number of the received votes is divided by a denominator and the quotients received are sorted in a descending order, where the highest values are the ones which receive a mandate.

The most largely used method is the one invented by the Belgian mathematician Victor D'Hondt.

Largest Remainder Methods, calculate the election quota, a.k.a. „Mandate Price", where the total number of votes is divided by the mandates to be distributed. After that the votes received for each candidate are divided by the quota and the integers received represent the number of the distributed mandates. The vacant mandates are distributed between the participants who have received the largest remainders during the step before.

The most popular method in that group is the Hare-Niemeyer method.

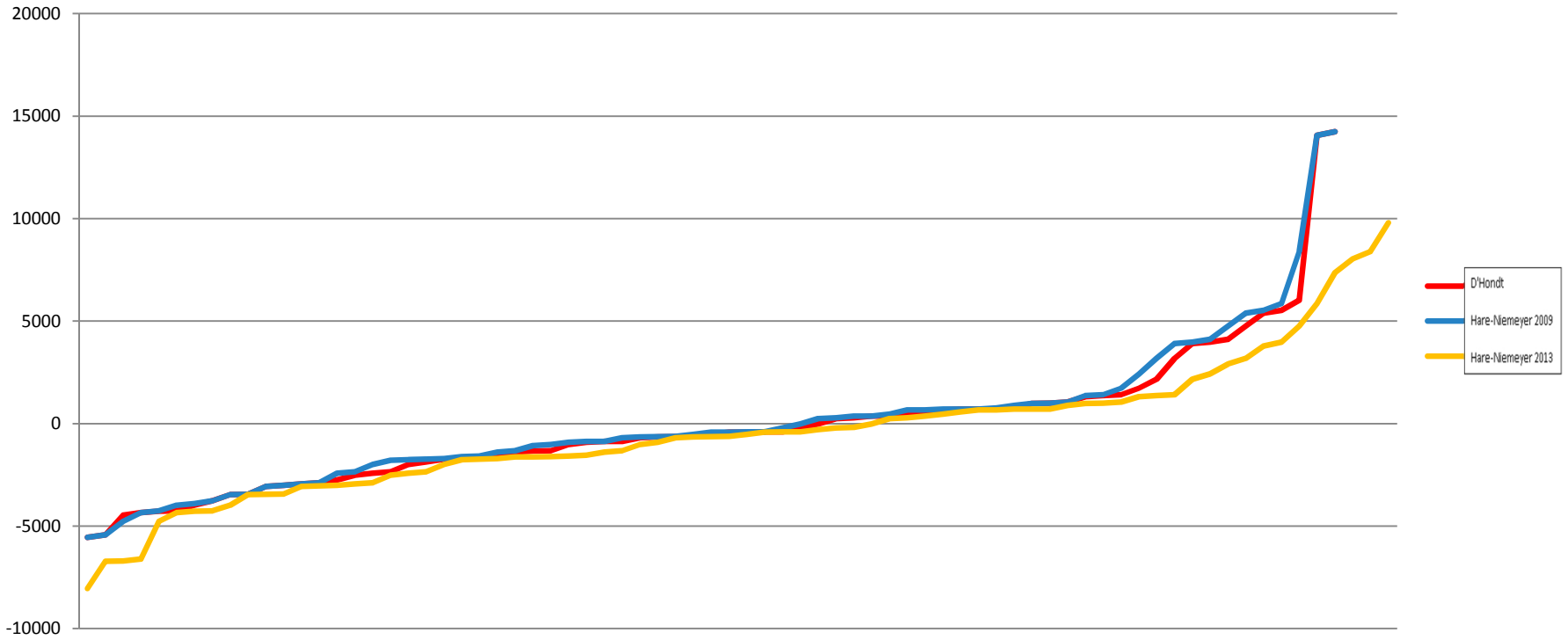
PROBLEM SOLUTION IN THE LAST 25 YEARS

As the sense of fairness is subjective, several terms for various scalar values which can be measured and compared shall be introduced. Below are presented the results of the four consecutive Parliamentary elections in Republic of Bulgaria within the period between 1989 and present day (in 1991, 2005, 2009 and 2014) , where the deviation is calculated using different methods.

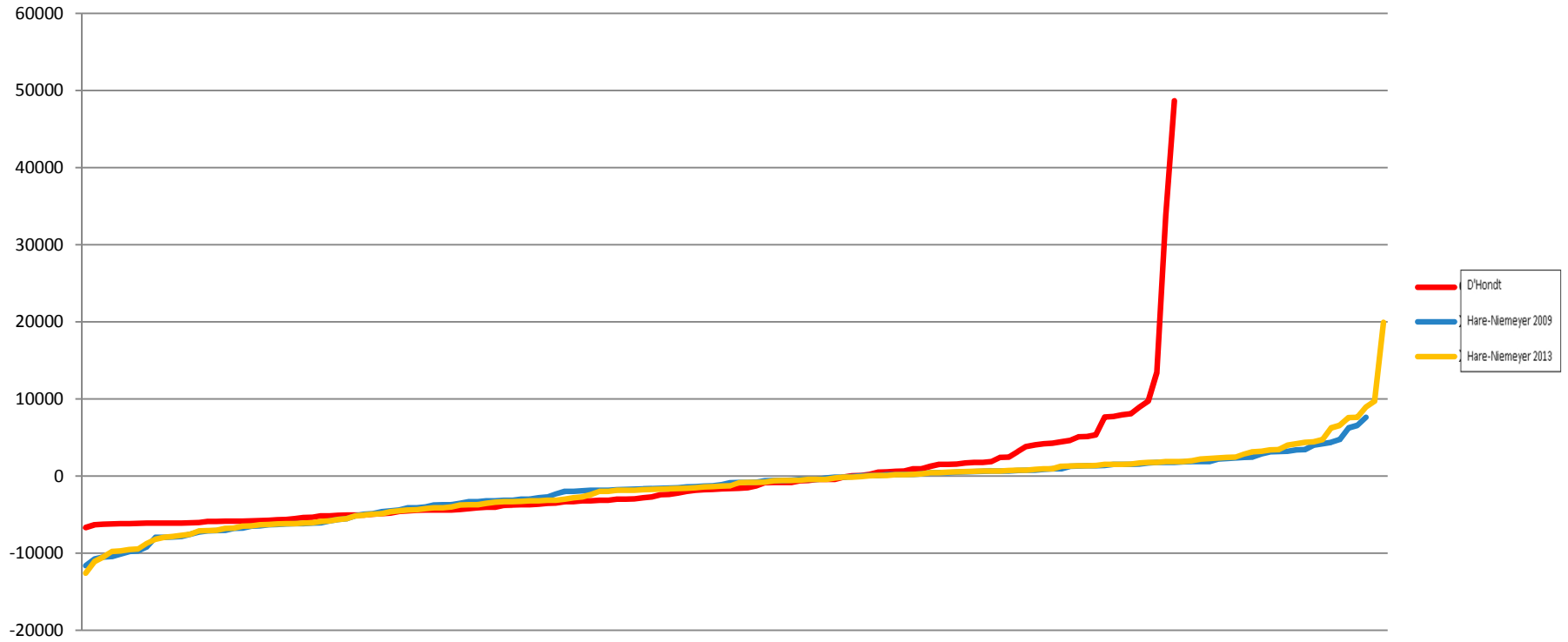
Three different calculation methods, applied by Central Election Commission, have been used accordingly:

- From 1991 to 2005 – D'Hondt method;
- In 2009 – Hare-Niemeyer method;
- From 2013 to 2014 – Hare-Niemeyer method (modified version).

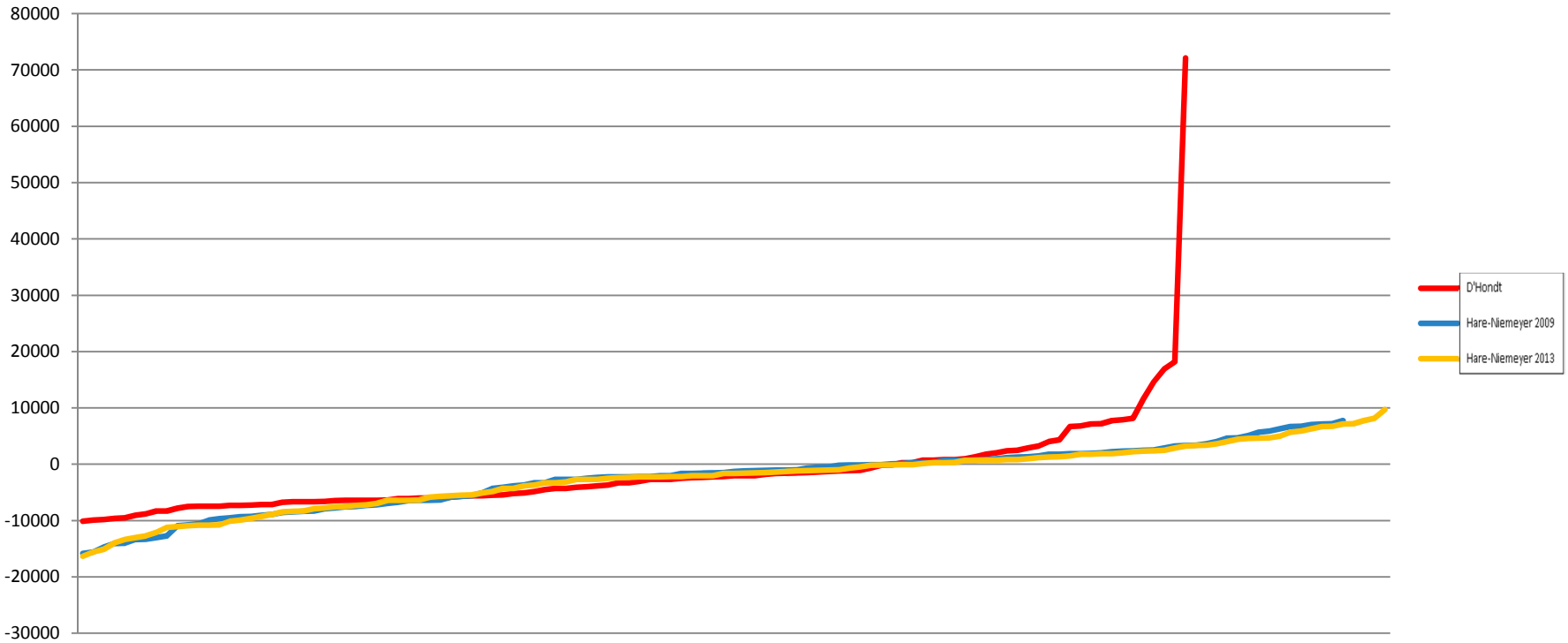
METHODS DEVIATIONS IN 1991



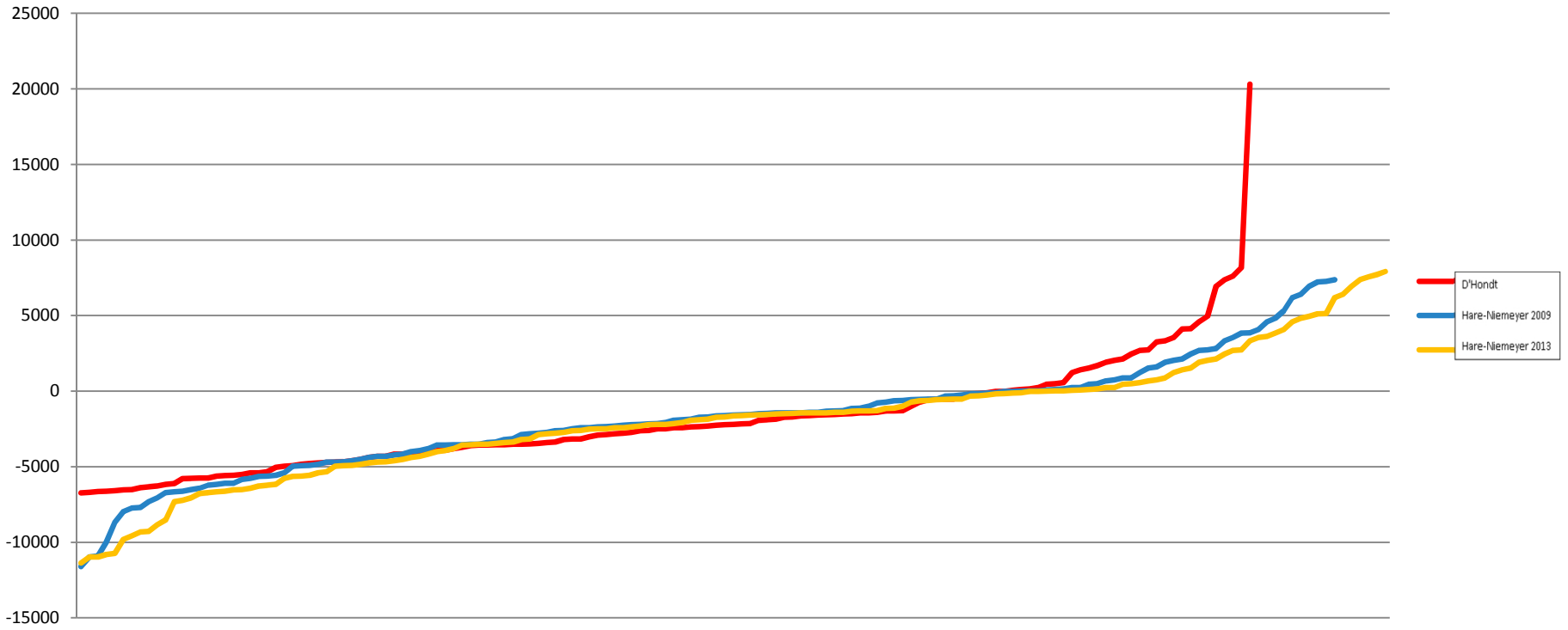
METHODS DEVIATIONS IN 2005



METHODS DEVIATIONS IN 2009



METHODS DEVIATIONS IN 2014



FACTORS: ELECTORAL CORRUPTION

Short definition of corruption is the following: “Abuse the social or governmental power with aim personal benefit”. The term “electoral corruption” is used to denote “the abuse of electoral institutions for personal or political gain”.

Types of electoral corruption:

- Manipulation of rules (legal framework);
- Manipulation of voters (preference-formation and expression);
- Manipulation of voting (electoral administration).

PROPOSED MATHEMATICAL MODEL

We propose a bi-criteria formulation, where a variable for the negative deviation ($\eta_i < 0$) and a variable for the positive deviation ($\rho_i > 0$) are introduced. Let we have P political parties and D districts. The criteria total deviation over all parties and total deviation over all districts have to be minimized simultaneously.

PROPOSED MATHEMATICAL MODEL

The proposed model for seat allocation is given by:

$$\min Z_1 = \sum_{i=1}^{P} |\eta_i| + \sum_{i=1}^{P} \rho_i \quad (1)$$

$$\min Z_2 = \sum_{j=1}^{D} |\eta_j| + \sum_{j=1}^{D} \rho_j \quad (2)$$

subject to

$$\left(\frac{s_i}{S}\right) 100 + \eta_i + \rho_i = \left(\frac{v_i}{V}\right) 100 \text{ for all } i = 1,\ldots,P; \quad (3)$$

$$\sum_{i=1}^{P} s_i = S \ , \ \sum_{j=1}^{D} s_j = S \quad (4)$$

$$s_i \geq 0 \text{ and integer}, \ s_j \geq 0 \text{ and integer}, \ j = 1,\ldots,D; \quad (5)$$

$$Z_1 \geq 0 \quad (6)$$

$$Z_2 \geq 0 \quad (7)$$

PROPOSED MATHEMATICAL MODEL

To solve the proposed set allocation problem (1)-(7) we introduce a scalarizing criterion

$$\min \omega_1 Z_1 + \omega_2 Z_1 \qquad (8)$$

replacing criteria (1) and (2). Here the coefficients ω1 and ω2 are subject to:

$$\omega_1 + \omega_2 = 1 \qquad (9)$$

and to

$$\omega_1 \geq 0, \omega_2 \geq 0 \qquad (10)$$

PROPOSED MATHEMATICAL MODEL

Then we solve the problem (8) subject to (3)-(7), (9)-(10) in the following manner:

- **Step 1.** Calculate the initial solution for seat allocation rounding off to the nearest integer the values of seats allocated to each party, obtained by means of Hare quota Hq.
- **Step 2.** The decision maker (DM) inputs values for the coefficients ω_1 and ω_2.
- **Step 3.** The problem is solved as mixed integer multiobjective one by the weighting method.
- **Step 4.** If the DM is not satisfied with the obtained solution he/she repeats the procedure starting it from the initial solution and setting new values for the coefficients ω_1 and ω_2: Go to Step 2. Otherwise go to Step 5.
- **Step 5.** Show the generated solutions to the DM, who chooses one of them as a best compromise solution. END.

CONCLUSIONS

- The elections are fight for power and a multitude of interests collide in them. Each of the participants is ready to use authorized and unauthorized methods for influencing the total elections outcome.
- Holding of honest and fair elections is among the key factors in ensuring a stable government throughout the elected ones' mandate. So it is important that the whole environment in which the elections are held to be adapted to the electoral sets to a maximum extent, so that minimum distortions in results to be ensured.

THANK YOU FOR YOUR ATTENTION !

IF YOU HAVE ANY QUESTIONS – I'M WELCOME TO ANSWER.

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ELECTORAL SYSTEMS AND FACTORS FOR VOTING RESULTS DISTORTION

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